Construction of Hierarchical Modular Structures with Geometrical Symmetry

- ZOME TOOL can visualize three-dimensional hierarchical modular structures -

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1. Introduction

It takes too much time and cost to construct large and complex space systems including the case of the International Space Station. During construction and operation of such structures, some changes of initial designs or schemes are inevitable. Reasons of the changes are not only technical or scientific but also social or political affairs. Therefore basic design principle for future large space structure systems needs adaptability to such various changes.

For such future systems, we proposed hierarchical modular structure systems inspired by hierarchical property of natural things including fractal geometry [1]. In this paper, we describe construction and practical visualization of hierarchical modular structures with geometrical symmetry. Using ZOME TOOL we can easily extend our concept to three dimensional structures. Our approach and some three dimensional examples made of ZOME TOOL are also demonstrated.

2. Hierarchical Modular Structure Systems

The proposed systems consist of a number of same-shaped modules which are hierarchically assembled. They can form various sizes and shapes with the same-shaped modules assemble by systematical rules. The rules are expressed by the following mathematical equations.

$$
\begin{aligned}
G^1 &= A^1\left(M, M, \cdots, M\right),\\
G^2 &= A^2\left(G^1, G^1, \cdots, G^1\right)\\
&\cdots\\
G^k &= A^k\left(G^{k-1}, G^{k-1}, \cdots, G^{k-1}\right)\\
&= A^k\left(A^{k-1}\left(G^{k-2}, \cdots, G^{k-2}\right), \cdots, A^{k-1}\left(G^{k-2}, \cdots, G^{k-2}\right)\right)\\
&\cdots,
\end{aligned}
\qquad (1)
$$

where M is an initial member, A^k is k th assembly rule to generate a next-generation structure, and G^k is a k th-generation structure. Assembly rule expressed by A^k may be different for each

k. This expression shows that one G^k is composed of some previous generation G^{k-1}s. In particular, a uniform A^k generate the structures with fractal properties.

3. Two dimensional hierarchical modular structures based on geometrical symmetry

It is rather easy to find dynamical equilibrium under free-free condition including in orbit for structure systems with geometrical symmetry. And it is easy to evaluate connective compatibility between adjacent modules or groups of modules. To construct such symmetric structures, an assembly rule based on geometrical symmetry is introduced into A^k in equation (1). For two dimensional structures, an initial member M is one dimensional element shown in Fig.1. An assembly rule based on geometrical symmetry generates a closed loop system using rotation mappings. Stiffness property of a closed loop system is rather higher. A mapping is characterized by a rotation with an angle $2\pi/n[rad]$ around a center on a symmetric axis. Examples of first generation structures made of one-dimensional elements are shown in Fig.1. According to this rule, arbitrary regular polygon can be constructed.

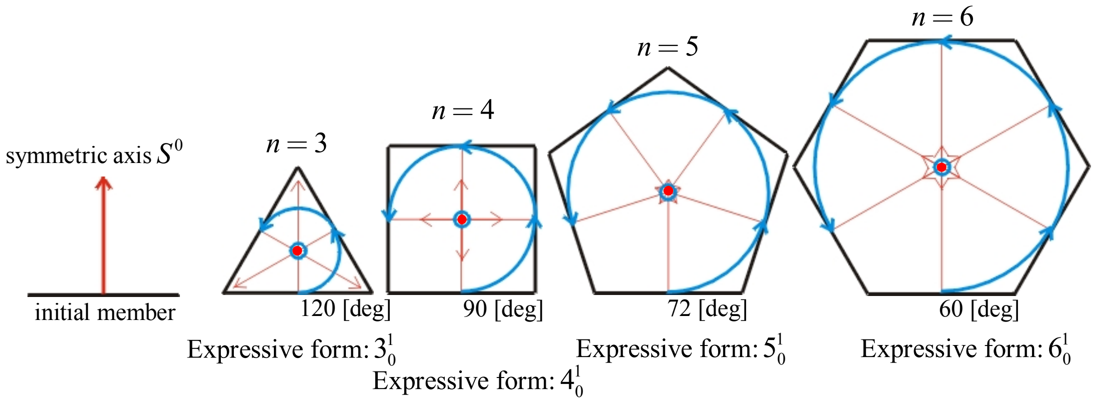


Fig.1 Examples of first generation "closed-loop" structures based on rotation mappings

As shown in Fig.1, we introduce an expressive form using a direction of a symmetric axis and a number of elements for a closed-loop. A superscript and subscript numbers indicate its generation k in equation (1), and a direction of a symmetric axis, respectively. Briefly, $n_\theta^k(\ ):k=2,3,\cdots$ means a rotation mapping with $2\pi/n[rad]$ around a fixed point on a $\theta[rad]$-direction symmetric axis, which make a k th

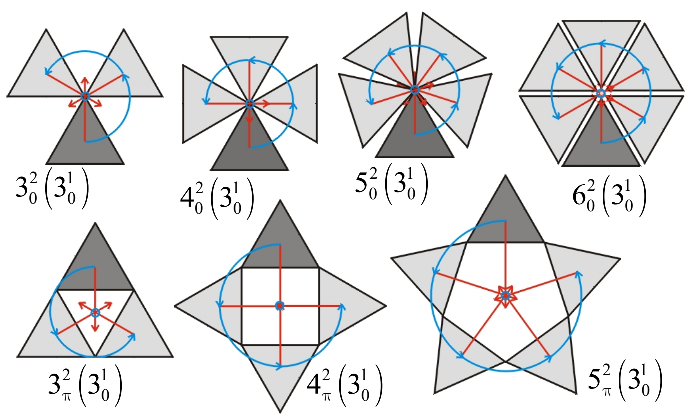


Fig.2 Second generations based on triangle modules

generation of n ($k-1$)-generation modules or groups of modules.

Applying rotation mappings to first generation modules, second generation structures can be constructed in a same way. Figure 2 illustrates some examples of second generations which consist of regular triangle modules. Their expressive forms are also shown. Clearly all constructed structures keep partial and total geometric symmetry. Figure 3 shows some examples of third generations which are generated by mappings based on same number elements. Especially $3^3_\pi\left(3^2_\pi\left(3^1_0\right)\right)$ is Sierpinski-Gasket, which is known for its fractal property. In a way other patterns in Fig.3 have fractal property, because each of them uses mappings with a same geometrical property.

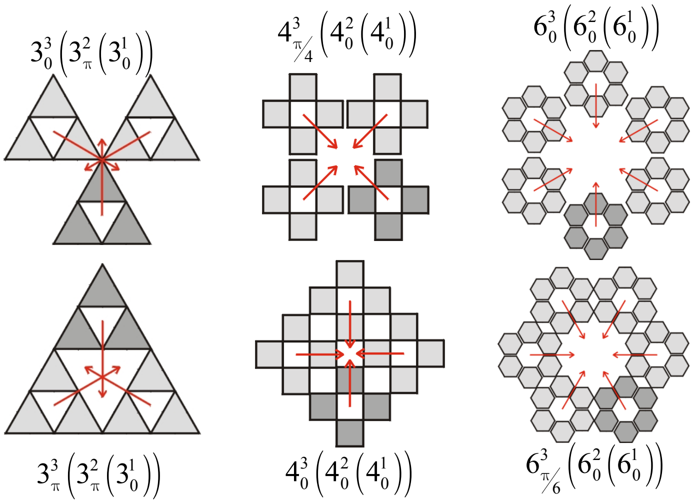


Fig.3 Examples with fractal properties

4. Three dimensional hierarchical modular structures based on geometrical symmetry

Rotation mapping in Euclidean two-dimensional space (SO(2)) might be extended into finite rotation in Euclidean three-dimensional space (SO(3)). In that case, assembly rules would be based on rotation group and a first generation module would be a polyhedron. Finite rotation group involves the three subgroups: cyclic group, dihedral group, and polyhedral group. Introducing various mapping based on these groups systematically provide various three-dimensional hierarchical modular structures. Expressive forms are used in the same way. But it is not easy to visualize them and evaluate geometrical consistency. ZOME TOOL can easily visualize these systematic structure systems.

Configurations using cyclic and dihedral groups are considered as extension of two-dimensional cases [1]. They include plane truss structures such as octahedral plane trusses shown in Fig.4.

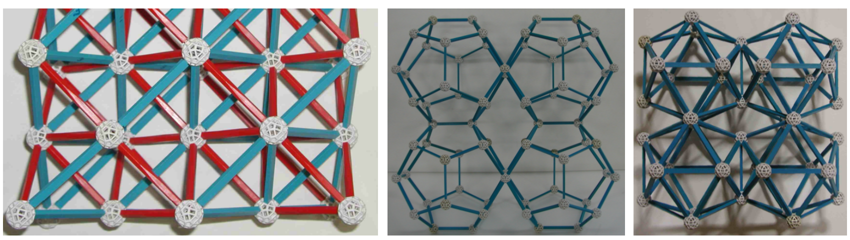


Fig.4 Examples generated by cyclic or dihedral mapping

In this paper, we address configurations using polyhedral subgroups. This concept easily

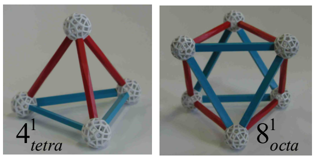
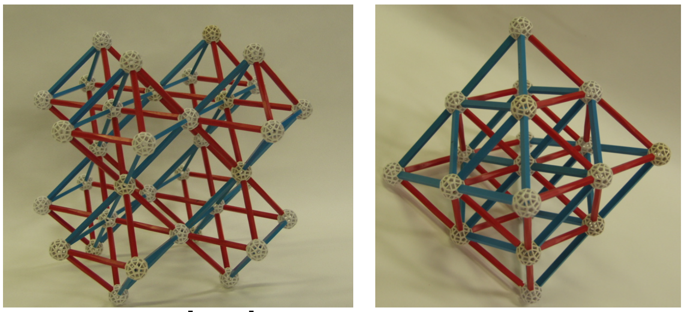


Fig.5 First generations

provides three-dimensional closed-loop systems. According to this construction rules, five regular polyhedral modules are first generations. Tetrahedral and octahedral modules and their expressive forms are shown in Fig.5. Upper and lower subscripts mean their generations and types of polyhedral mappings, respectively.

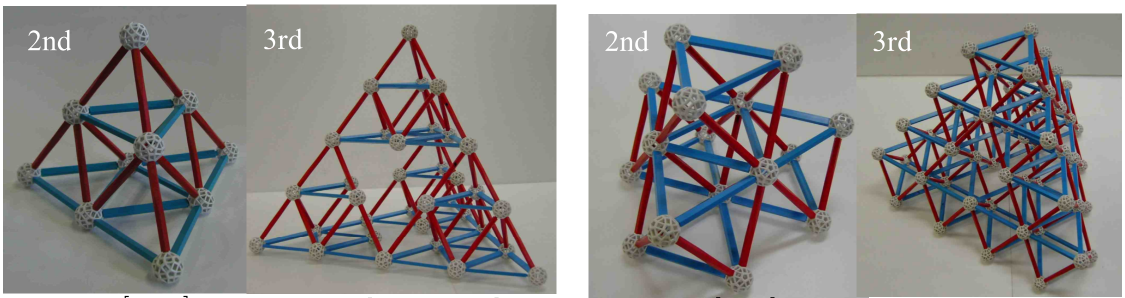
Figure 6 shows second generation examples based on octahedral first generation modules. Expression forms are also shown. In the case of Fig.6 (a), mappings for first and second generations are different. In the case of Fig.6 (b), mappings for first and second generations are same. The latter has so-called fractal property.



(a) $4^{2}_{tetra}\left[8^{1}_{octa}\right]$ (b) $8^{2}_{octa}\left[8^{1}_{octa}\right]$

Fig.6 Second generations based on octahedral modules

Second and third generations based on tetrahedral and octahedral modules are shown in Fig.7.



$4^{2}_{tetra}\left[4^{1}_{tetra}\right]$ $4^{3}_{tetra}\left[4^{2}_{tetra}\left[4^{1}_{tetra}\right]\right]$ $4^{2}_{tetra}\left[8^{1}_{octa}\right]$ $4^{3}_{tetra}\left[4^{2}_{tetra}\left[8^{1}_{octa}\right]\right]$

(a) first generation: tetrahedron 4^{1}_{tetra} (b) first generation: octahedron 8^{1}_{octa}

Fig. 7 Second and third generations based on tetra- and octa-hedral modules

It is clear that outline of whole structures are strongly related to the last mapping. Figure 7(a) demonstrates so-called three-dimensional Sierpinski-Gasket truss, which is a one of famous shapes with fractal property. Forth generation examples are shown in Fig.8. In this way, we can systematically construct three-dimensional hierarchical modular structures with geometrical symmetry. To evaluate geometrical consistency, we proposed construction method using mapping matrix shown in Fig.9 [2].

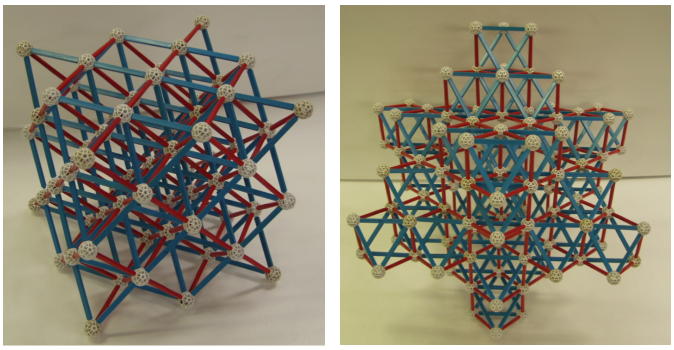


Fig.8 Forth generation examples

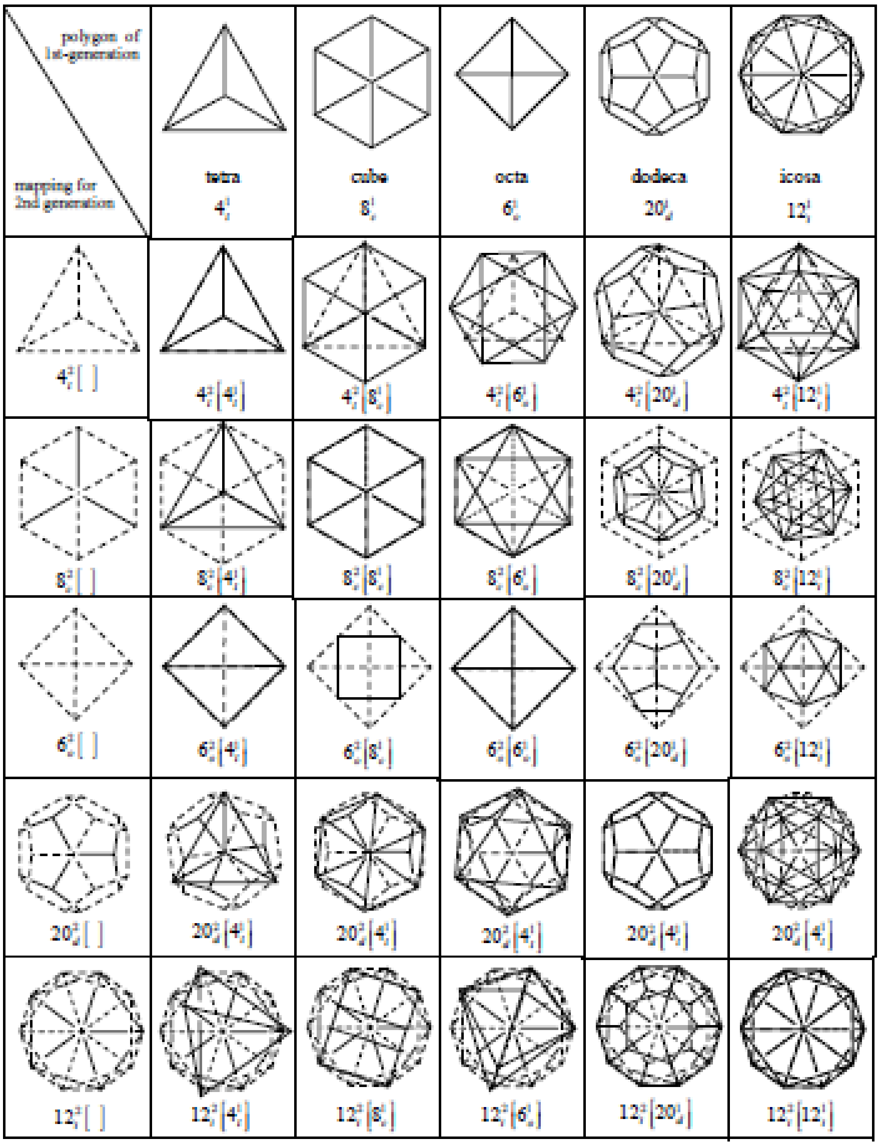


Fig. 9 Mapping matrix for construction method using polyhedral group

5. Conclusion

This paper describes construction of hierarchical modular structures with geometrical symmetry. Examples by ZOME TOOL can easily visualize three-dimensional structure systems.

Reference

[1]N.Kishimoto, M.C.Natori, "Hierarchical Modular Structures and Their Geometrical Configurations for Future Large Space Structures", International Symposium on Shell and Spatial Structures from Models to Realization (IASS2004), Montpellier, France, Sep.20-24, 2004, TP013.

[2]N.Kishimoto, Research on Structure Systems Based on Hierarchical Concepts, Ph.D Thesis of Tokyo University, 2004.